

WORKING ON ALGEBRAIC THINKING

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A STARTING POINT

At the first meeting of the CERME Algebra Group in Larnaca, Cyprus all participants were asked to write down a sentence starting with “At the beginning of the work of the Algebra Group I think that algebraic thinking *must* include...”, and to keep it safe till the last session. Then we jumped to work in several problems proposed by John Mason and Janet Ainley, to begin our work by experiencing the kind of thinking we have to reflect on, and by interacting among ourselves.

Before meeting in Larnaca we had gone through the process of peer reviewing the papers, ending with 14 papers accepted for presentation.

These papers were presented in four themed groups, which organised the type of research or theoretical questions dealt with, under the labels:

- expressing generality,
- teachers,
- expressions and equations in the interface between arithmetic and algebra, and
- mathematical objects and representations.

After four sessions discussing the presented papers, we moved to discussion around two groups of issues that emerged:

- Object and meaning / situated algebra
- Emergent algebabble / Generality as central / generalisation, abstraction, formalisation

OBJECT AND MEANING / SITUATED ALGEBRA

The papers more relevant to this topic were those by Bagni; Barbosa, Palhares and Vale; Dörfler; Fischer; Lagrange and Chiappini; Mason; Wilson and Ainley; Rahman. They raised issues concerned in various ways with algebraic activities situated in the use of artefacts and representations used by teachers and learners.

Our discussion went through a set of questions on the emergence and extension of meaning, and on the characteristics of the situations in which meaning emerge.

- How can teachers extend situated meanings?
- Can meaning emerge from the practice of manipulation (of symbols or objects) with rules?
- Can meaning emerge without practice?

- What features of a situation do we anticipate can afford access beyond the situation?

The term ‘situation’ was used broadly to encompass task, medium, pedagogic approach by teacher, and classroom ethos, and we pointed out the central role of the teacher in going beyond it:

- Teacher stressing purpose.
- Teacher’s choice of emphasis in original situation.
- Teacher referring back from new context to original situation.

EMERGENT ALGEBABBLE / GENERALITY AS CENTRAL / GENERALISATION, ABSTRACTION, FORMALISATION

The papers more relevant to this topic were those by Alexandrou-Leonidou and Philippou; Drury; Dörfler; Fisher; Gómez; Hadjidemetriou; Mason; Molina, Castro and Mason; Papaieronymou. They raised issues concerned in various ways with expressing generality or symbol manipulation or problem solving as the core principle of school algebra, and with pupils trying to make sense of the algebraic language they are learning.

Curricular issues

Discussion about the central role of generalisation in the teaching and learning of school algebra led to the question *what is school algebra about?*, and the discussion of curricular issues.

Generalisation was contrasted? confronted with symbol manipulation and with problem solving, as the core principle of the curriculum of school algebra.

One position was that algebraic expressions can be used as tools or as objects. The approach is then to use them as objects in their own right, and use manipulation of expressions to investigate expressions – what can I do with them? Algebraic expressions are diagrams in Peirce’s sense, and these diagrams are the very objects of mathematical activities.

Another position stated that a curriculum organised around problem solving does not exclude, but includes expressing generality, at least in two ways. First of all expressing generality is a problem solving activity, hence if the core principle of the curriculum is problem solving, this does not have to mean that pupils are presented only with work on quantitative arithmetic-algebraic word problems, but it should also include working with problems in which the aim is to express generality.

Secondly, expressing generality is a way to endow algebraic expressions with meaning. In order to solve word problems by using the algebraic language, pupils need to learn the use of algebraic language in a meaningful way, its syntax and the special feature this language has of calculating with the expressions without resorting

to its content – a possibility due to the fact that algebraic expressions are icons (or diagrams), in Peirce’s terms.

“Meaningless symbol manipulation”, or symbol manipulation following a set of conventional rules, can come later on, when one can set aside the meaning of expressions to carry on the calculations. Actually, rules of symbol manipulation are also meaningful. Ways of symbolising can be social conventions, but rules of manipulation of symbolic expressions are grounded in the observation of some structure.

What does ‘meaningless symbol manipulation’ mean? Setting aside meaning – but can also mean without a sense of overall purpose.

Historically, the purpose of symbol manipulation, and of solving equations is the solutions of (classes) of problems. Through a process of progressive abstraction algebraic expressions are studied as objects, and so on. Mathematisation is a practice of progressive abstraction. To jump into one level, i.e. symbol manipulation, instead of climbing up levels, or to be thrown into one level instead of being given the opportunity of climbing up the staircase of levels is not a good way to organise the curriculum.

On abstraction

Generalisation was contrasted with abstraction, by pointing out that abstraction could consist of

- taking a property – forgetting the object and asking ‘what else has this property?’;
- forgetting some meanings;
- transforming - a means of organisation of objects into a single object.

On idiosyncratic signs

Algebabble is an expression used to capture what pupils do when they are going through the process of giving meaning to algebraic activity. The idiosyncratic signs they produce are

- windows to pupils cognitions,
- endowed with meaning by them.

A PRODUCT OF OUR DISCUSSION

At the last meeting of the CERME Algebra Group in Larnaca, Cyprus all participants were asked to produce the paper in which they have written their ideas on what algebraic thinking *must* include, and to reflect on what new things they could see in their papers after our discussions. One of them just said: “looking at my paper what I see is if students are able to see the structure of an equality they have begun algebraic thinking”.