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Chapter 3

# Integrating history: research perspectives

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*Abstract*: The question of judging the effectiveness of integrating historical resources into mathematics teaching may not be susceptible to the research techniques of the quantitative experimental scientist. It is better handled through qualitative research paradigms such as those developed by anthropologists.

## 3.1 Introduction

Over the past twenty years or so there has been a growing interest in history by teachers and educators. What consequences may this interest have for mathematics education? And how can we judge its effectiveness? A great many articles have appeared in increasing number over this time, including educational reports, reflections of teachers and accounts of teaching experiences. This material gives different arguments in favour of including a historical dimension in the teaching of mathematics, and often contains reasons for why the teacher believed it to be effective. We also can identity through this material different ways in which it is effective, depending for example upon whether the presence of history is implicit or explicit in the teaching situation; and whether the use of history is local, being used for a particular topic, or global–that is, characterising the didactic strategy or the way the mathematics is taught.

The two most commonly presented reasons for the inclusion of a historical dimension are that history of mathematics provides an opportunity for developing our view of what mathematics is; and that it allows us to have a better understanding of concepts and theories. In each of these there is a sequence of developing understandings: the history of mathematics can first change the teacher's own perception and understanding about mathematics, then it will influence the way mathematics is taught, and finally it affects the way the student perceives and understands mathematics. We can evaluate the effectiveness of introducing an historical dimension into the teaching of mathematics through an examination of this process.

The breadth of the arguments is such that we cannot approach the question of using history of mathematics in a quantitative or piece-meal fashion. We offer examples later of case-study evaluations that use a holistic and qualitative approach. Our approach should not be seen as prescriptive: we do not propose models or programmes. On the contrary, a view of the whole of the process suggests we should be cautious; there are limits and risks attached to an approach that takes too simplistic a view of the significance of history in mathematics education.

The change which this may bring about in the image of mathematics held by the teacher can be presented as a contrast between a formal presentation of mathematics and a heuristic approach

provided by history. This difference corresponds to a contrast in pedagogic style: that of the traditional teacher, where knowledge is handed out by the teacher, and a learning process based on mathematical activity by the student. The heuristic view is associated with a constructivist view of mathematics in which knowledge is constructed step by step and concepts are clarified through solving new problems. History here is not only a revelation but also a source of reflection for the teacher, as is shown in the examples given in sections 3.3 and 3.4.

The historical dimension encourages us to think of mathematics as a continuous process of reflection and improvement over time, rather than as a defined structure composed of irrefutable and unchangeable truths. The latter view is one that may be held by the teacher fresh from college or university and without experience of research. Thinking about mathematics as an intellectual activity, rather than as a finished product, means thinking of problems to be solved, of the importance of conjectures and the value of intuition. In this sense, the pupil in mathematics and the mathematical researcher are engaged in the same activity. The historical dimension here can bring about a global change in a teacher's approach, whether or not the historical element is explicitly present in the classroom. Historical knowledge helps the teacher to understand stages in learning as well as to propose problems inspired by history. It is interesting to note that teachers in some countries are tempted to contrast the image of mathematics which history presents with that given by the 'modern mathematics' reforms which were popular in the 1960s. Under modern mathematics reforms the teaching of mathematics began with the most recent formulation of concepts of mathematics, which is the exact opposite of the historically-informed approach.

Historical awareness also leads teachers to change the way they think about their students. As shown in sections 3.5 and 3.6, the responses students make to an historical problem take on a new character when they are compared with the responses made by mathematicians through the ages. Historical and epistemological analysis helps the teacher to understand why a certain concept is difficult for the student and can help also in the teaching strategy and development. This has two particular consequences for how the teacher can use the historical dimension effectively. First, the teacher can adopt a constructive attitude towards the errors the students make. Secondly, the teacher can focus on producing a variety of responses to a given problem; relate them to what the students know or to the connections within their present knowledge. The historical dimension leads to the idea that mathematics is no longer a sequence of discrete chapters (in geometry, algebra or analysis), but is an activity of moving between different ways of thinking about mathematical concepts and tools.

When we learn about the historical development of mathematics it affects how we think about the time our students spend in developing mathematical understanding. If it took several centuries for mathematicians to be able in make explicit our current concept of a limit, for example, it is going to take a considerable time for our students as well. There is time needed, also, to deal with the epistemological problems inherent in manipulating the infinite. And then it takes time to move from the idea of the limit as a tool for solving problems to the idea of the limit as part of an integrated body of mathematical knowledge linked to other concepts, such as that of real number or set. We should note, however, that even if students are led to construct their knowledge in a way that parallels the historical development, it does not mean that there will be an exact match between the student's construction and the historical sequence. After all, obstacles encountered by mathematicians in history may not be those that face the student of today. Nonetheless, learning that there were obstacles is in itself beneficial.

If the teacher decides to introduce history explicitly in class, it can be done either as part of a global approach in terms of a didactic strategy or in a local way, in the context only of teaching a particular topic. In addition to the points made above, the teacher may wish to provide a cultural context for mathematical knowledge by locating this knowledge within the history of mankind and ideas. Where explicit use of history is concerned, there are limitations and risks. It is seen in section 3.7 that it can be difficult to understand the procedure used by a mathematician of ancient times if it is not set within the historical context. There is a difficulty here for the teacher to resolve, well before it becomes one for the student (this raises the question of the training of teachers). At least two types of danger can arise when using history explicitly. First, using piecemeal historical illustrations can give a false and truncated view of what mathematics, and indeed history, was really like historically. Alternatively, in trying to present a global historical view, we could be in danger of ending up with an education in mathematics history quite independent of the needs of mathematics education. At worst, one could fear that mathematics might one day be replaced by a teaching of its history.

It is therefore a question of integrating history *within* the teaching of mathematics, and that is why teachers talk of a historical dimension, a historical style, or a historical perspective in mathematics education. These terms describe, in a general way, the teacher's active mobilisation of all his or her historical and epistemological reflections. In evaluating the effectiveness of using history in mathematics classrooms, we have to consider all the aspects of a historical dimension. It is possible to appreciate the effectiveness of using history through an ethnographic approach to examples of practice. Section 3.2 suggests we should proceed by an analysis of case studies, using the observations of participants and interviews with students and teachers, and drawing on existing written accounts, in particular on articles where teachers explain why and how their historical approach to mathematics has changed the way they teach.